Maths With Zombies

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22. The Recruitment Problem

There's a zombie outbreak in a city of 15 million people and the army has been ordered to deal with it, and as the general that means you have to decide what to do. You could send in all your troops, but then you would have none spare if there's another outbreak somewhere else. Instead, you decide to send in a skeleton force of just 1,600 soldiers with orders to recruit members of the public to help them fight the zombies. It's a novel solution, and leaves you with plenty of men in case there are any other outbreaks, but no one else thinks it'll work. To prove them wrong, you set out to do the maths. You know that each soldier can hunt down and kill twenty zombies a day. In the evening, each soldier can also recruit one new soldier from the people in the city who will join the fight the following day. However, each night the zombies will fight back and infect five normal people who then become zombies. You know there's currently 50,000 zombies in the city. Have you made the right decision?

A. Yes, the maths shows this is the right decision. It might take a while, but the strategy will work and the soldiers will regain control of the city.

B. The maths shows that while the strategy is sound, you'd need to send in more troops at the start for it to be successfully implemented.

C. No. The maths shows that this is the wrong decision. No matter how many soldiers I sent in, my strategy would never work. This is because the number of zombies keeps increasing at a rate faster than the soldiers can kill them. The only thing which can stop the outbreak spreading is to nuke the entire city before the situation gets any worse.

What answer did you get?

A: Your maths must have been wrong. If you send in only 1,600 soldiers, you'll never managed to get the outbreak under control and the city will be lost within days.

B: Spot on! Just as well you did the maths, or you would have sent in too few troops. In fact, while an initial force of 1,600 men cannot get the outbreak under control by following your strategy, sending in just 67 more in your initial force would ensure it worked.

C: Whoa, I don't know what happened with your calculations there, but you're way off. Nuking the city will certainly get the outbreak under control, but it'll also needlessly kill a lot of innocent people. Maybe you should check your figures before you take quite such a drastic decision.

How to work it out: The maths here is quite complicated, but it reveals something very interesting. To work out the number of zombies at the start of each day, you need to know how many zombies and soldiers there were at the start of the day before. If you know this, you can use the formula Nt+1 = (Nt - (St * 20))*5 to calculate the number of zombies on at the start of any given day. In this formula, St is the number of soldiers at the start of the day before while Nt is the number of zombie at the start of the preceding day. Nt+1 is the number of zombies there will be at the start of the day itself. The value of 20 is the number of zombies which each soldier will kill during each day, while 5 is the number of people which each zombie will infect each night. Using this formula, you can work out what effect sending in different numbers of troops will have. Start by calculating the number of zombies at the start of day two of the campaign (remembering that the number of soldiers will double each day because of the new recruits) and then repeat this for days 3 to 10. If the number of zombies at the start of a day ever becomes zero, then the zombie outbreak will have been extinguished. If it doesn't, it hasn't.

If you plug in the initial number of soldiers (1,600) and zombies (50,000) into this formula, you'll see that the number of zombies quickly spirals out of control, reaching a whopping 17,088,000 by the start of day six, or more than the entire population of the city. This might make it seem like this strategy would never work, but this isn't true. For example, if you start in 1,700 soldiers and repeat the calculations, you'll see that the zombie problem gets sorted in just four days. In fact, the difference between success and failure comes down to just one single soldier. If you send in 1,666 soldiers, the city will be over-run by zombies by the start of day nine of your campaign. In contrast, if you send in 1,667 soldiers, the last zombie will be killed on day eight. This means the 1,667th soldier is an example of what Malcolm Gadwall calls a tipping point, where small changes can have big impacts on the final outcome of certain events. It is also an example of a chaotic system where small changes in the starting values can result in very different outcomes further down the road.